Representing Concepts in Time*

Martin Raubal

Department of Geography, University of California, Santa Barbara
5713 Ellison Hall, Santa Barbara, CA 93106, U.S.A.
raubal@geog.ucsb.edu

Abstract. People make use of concepts in all aspects of their lives. Concepts are mental entities, which structure our experiences and support reasoning in the world. They are usually regarded as static, although there is ample evidence that they change over time with respect to structure, content, and relation to real-world objects and processes. Recent research considers concepts as dynamical systems, emphasizing this potential for change. In order to analyze the alteration of concepts in time, a formal representation of this process is necessary. This paper proposes an algebraic model for representing dynamic conceptual structures, which integrates two theories from geography and cognitive science, i.e., time geography and conceptual spaces. Such representation allows for investigating the development of a conceptual structure along space-time paths and serves as a foundation for querying the structure of concepts at a specific point in time or for a time interval. The geospatial concept of 'landmark' is used to demonstrate the formal specifications.

Keywords: Conceptual spaces, time geography, concepts, representation, algebraic specifications.

1 Introduction

Humans employ concepts to structure their world, and to perform reasoning and categorization tasks. Many concepts are not static but change over time with respect to their structure, substance, and relations to the real world. In addition, different people use the same or similar concepts to refer to different objects and processes in the real world, which can lead to communication problems. In this paper, we propose a novel model to represent conceptual change over time. The model is based on a spatio-temporal metaphor, representing conceptual change as movement along space-time paths in a semantic space. It thereby integrates conceptual spaces [1] as one form of conceptual representation within a time-geographic framework [2].

Formal representations of dynamic concepts are relevant from both a theoretical and practical perspective. On the one hand, they allow us to theorize about how people's internal processes operate on conceptual structures and result in their alterations over time. On the other hand, they are the basis for solving some of the current pressing research questions, such as in Geographic Information Science (GIScience) and

* This paper is dedicated to Andrew Frank, for his 60th birthday. He has been a great teacher and mentor to me.

C. Freksa et al. (Eds.): SC 2008, LNAI 5248, pp. 328–343, 2008.

the disciplines concerned with ontologies. In GIScience, questions addressing which geospatial concepts exist, how to trace their developmental patterns, model their interactions (such as merging), and how to represent and process them computationally are of major importance [3]. Research on ontologies has focused on dynamic ontologies[1] for services to be integrated within the semantic web [4]. If we consider ontologies as explicit specifications of conceptualizations [5], then formal representations of dynamic concepts can be utilized for translation into ontologies.

Section 2 presents related work regarding concepts, and introduces conceptual spaces and time geography as the foundations for the proposed model. In Section 3, we define our use of representation and describe the metaphorical mapping from time-geographic elements to entities and operations in semantic space. We further elaborate on the difference of changes *within* and *between* conceptual spaces. Section 4 presents a computational model of conceptual change in terms of executable algebraic specifications. Within this model, the mappings of entities and operations are specified at the level of conceptual spaces, which consist of quality dimensions. Section 5 applies the formal specifications to represent the change of a person's geospatial concept of 'landmark' over time. The final section presents conclusions and directions for future research.

2 Related Work

This section starts with an explanation of the notion of *concepts* and their importance for categorization. We then introduce conceptual spaces and time geography as the underlying frameworks for representing concepts in time.

2.1 Concepts

There are several conflicting views on concepts, categories, and their relation to each other across and even within different communities. From a classical perspective, concepts have been defined as *structured mental representations* (of classes or individuals), which encode a set of necessary and sufficient conditions for their application [6]. They deal with what is being represented and how such information is used during categorization [7]. Barsalou et al. [8] view concepts as mental representations of categories and point out that concepts are context dependent and situated. For example, the concept of a chair is applied locally and does not cover all chairs universally. From a memory perspective, "*concepts* are the underlying knowledge in long-term memory from which temporary *conceptualizations* in working memory are constructed." [8, footnote 7] It is important to note the difference between concepts and categories: a concept is a mental entity, whereas a category refers to a set of entities that are grouped together [9].

Concepts are viewed as dynamical systems that evolve and change over time [8]. New sensory input leads to the adaptation of previous concepts, such as during the interactive process of spatial knowledge acquisition [10]. Neisser's [11] perceptual cycle is also based on the argument that perception and cognition involve *dynamic*

[1] See, for example, http://dynamo.cs.manchester.ac.uk/

cognitive structures (schemata in his case rather than explicit concepts). These are subject to change as more information becomes available.

Here, we use concepts within the paradigm of cognitive semantics, which asserts that meanings are mental entities—mappings from expressions to conceptual structures, which refer to the real world [12-14]. The main argument is therefore that a symbolic representation of an object cannot refer directly to objects, but rather through concepts in the mind. This difference between objects, concepts, and symbols is often expressed through the semiotic triangle [15].

2.2 Conceptual Spaces

The notion of *conceptual space* was introduced as a framework for representing information at the conceptual level [1]. Such representation rests on the before-mentioned foundation of cognitive semantics. Conceptual spaces can be utilized for knowledge representation and sharing, and support the paradigm that concepts are dynamical systems [16]. Sowa [17] argued that conceptual spaces are a promising geometrical model for representing abstract concepts as well as physical images. Furthermore, conceptual spaces may serve as an explanatory framework for results from neuroscientific research regarding the representational structure of the brain [1].

A conceptual space is a set of quality dimensions with a geometrical or topological structure for one or more domains. Domains are represented through sets of integral dimensions, which are distinguishable from all other dimensions. For example, the color domain is formed through the dimensions hue, saturation, and brightness. Concepts cover multiple domains and are modeled as n-dimensional regions. Every object or member of the corresponding category is represented as a point in the conceptual space. This allows for expressing the similarity between two objects as the spatial distance between their points. Recent work has focused on representing actions and functional properties in conceptual spaces [18].

In [19], a methodology to formalize conceptual spaces as vector spaces was presented. Formally, a conceptual vector space is defined as $\mathbf{C}^n = \{(c_1, c_2, \ldots, c_n) \mid c_i \in \mathbf{C}\}$ where the c_i are the quality dimensions. A quality dimension can also represent a whole domain and in this case $c_j = \mathbf{D}^n = \{(d_1, d_2, \ldots, d_n) \mid d_k \in \mathbf{D}\}$. Vector spaces have a metric and therefore allow for the calculation of distances between points in the space. This can also be utilized for measuring distances between concepts, either based on their approximation by 'prototypical points' or 'prototypical regions' [20]. In order to calculate these *semantic distances* between instances of concepts all quality dimensions of the space must be represented in the same relative unit of measurement. Assuming a normal distribution, this is ensured by calculating the z scores for these values, also called z-transformation [21]. For specifying different contexts one can assign weights to the quality dimensions of a conceptual vector space. This is essential for the representation of concepts as dynamical systems, because the salience of dimensions may change over time. $\mathbf{C}^n$ is then defined as $\{(w_1c_1, w_2c_2, \ldots, w_nc_n) \mid c_i \in \mathbf{C}, w_j \in \mathbf{W}\}$ where $\mathbf{W}$ is the set of real numbers.

2.3 Time Geography

People and resources are available only at a limited number of locations and for a limited amount of time. Time geography focuses on this necessary condition at the core of human existence: "How does my location in space at a given time affect my ability to be present at other locations at other times?" It defines the space-time mechanics by considering different constraints for such presence—the capability, coupling, and authority constraints [2]. The possibility of being present at a specific location and time is determined by people's ability to trade time for space, supported by transportation and communication services.

Space-time paths depict the movement of individuals in space over time. Such paths are available at various spatial (e.g., house, city, country) and temporal granularities (e.g., decade, year, day) and can be represented through different dimensions. Figure 1 shows a person's space-time path during a day, representing her movements and activity participation at three different locations. The tubes depict *space-time stations*—locations that provide resources for engaging in particular activities, such as sleeping, eating, and working. The slope of the path represents the travel velocity. If the path is vertical then the person is engaged in a stationary activity.

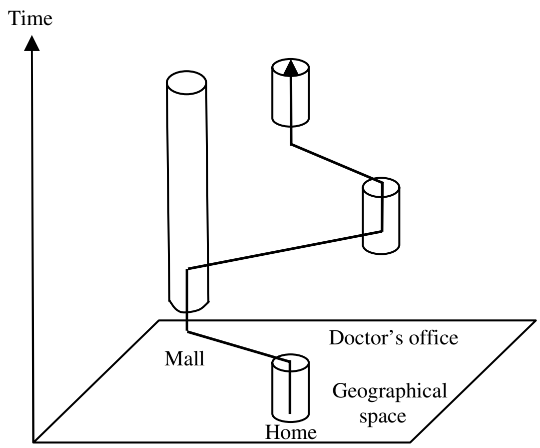

Fig. 1. Space-time path of a person's daily activities

Three classes of constraints limit a person's activities in space and time. *Capability constraints* limit an individual's activities based on her abilities and the available resources. For example, a fundamental requirement for many people is to sleep between six and eight hours at home. *Coupling constraints* require a person to occupy a certain location for a fixed duration to conduct an activity. If two people want to meet at a Café, then they have to be there at the same time. In time-geographic terms, their paths cluster into a space-time bundle. Certain domains in life are controlled through *authority constraints*, which are fiat restrictions on activities in space and time. A person can only shop at a mall when the mall is open, such as between 10am and 9pm.

All space-time paths must lie within *space-time prisms* (STP). These are geometrical constructs of two intersecting cones [22]. Their boundaries limit the possible locations a path can take based on people's abilities to trade time for space. Figure 2 depicts a space-time prism for a scenario where origin and destination have the same location. The time budget is defined by $\Delta t = t_2 - t_1$ in which a person can move away from the origin, limited only by the maximum travel velocity. The interior of the

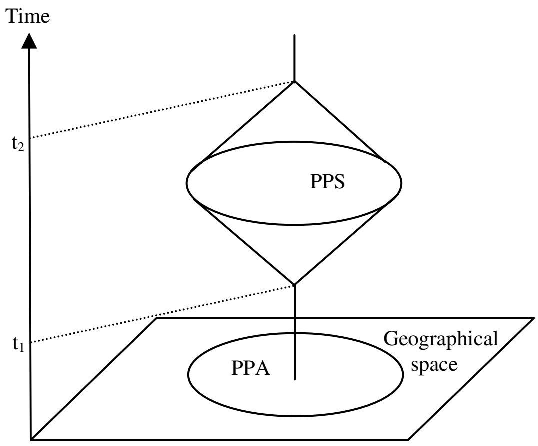

Fig. 2. Space-time prism as intersecting cones

prism defines a *potential path space* (PPS), which represents all locations in space and time that can be reached by the individual during Δt. The projection of the PPS onto geographical space results in the *potential path area* (PPA) [23].

3 A Spatio-temporal Metaphor for Representing Concepts in Time

In this section, we first give a definition of representation, which applies to the model presented here. The metaphorical mapping from time-geographic to semantic-space elements is then explained. A formal model for the resulting semantic space will be developed in the next section.

3.1 Representational Aspects

Different definitions of what a representation is have been given in the literature. In this paper, we commit to the following: "A world, *X*, is a representation of another world, *Y*, if at least some of the relations for objects of *X* are preserved by relations for corresponding objects of *Y*." [24, p.267] In order to avoid confusion about what is being represented how and where regarding conceptual change over time, we distinguish between two representations—the *mental world* and the *mental model*—, according to [24]. The mental world is a representation of the real world and concerned with the inner workings and processes within the brain and nervous system (i.e., inside the head). Here, we formally specify a possible mental model as a representation of the mental world[2]. The goal is to be able to use this model to explain the processes that lead to the change of concepts in time. In this sense, we are aiming for informational equivalence [24], see also [25] and [26] for examples from the geospatial domain.

[2] A mental model is therefore a representation of a representation of the real world—see Palmer [24] for a formal demonstration of this idea.

3.2 Metaphorical Mapping

The proposed mental model for representing conceptual change in time is based on a spatio-temporal metaphor. The power of spatial metaphors for modeling and comprehending various non-spatial domains has been widely demonstrated [27-30]. From a cognitive perspective, the reason for such potential is that space plays a fundamental role in people's everyday lives, including reasoning, language, and action [31].

Our representation of conceptual change in a mental model is based on the metaphorical projection of entities, their relations, and processes from a *spatio-temporal source domain* to a *semantic target domain*. As with all metaphors, this is a partial mapping, because source and target are not identical [30]. Concepts are represented as n-dimensional regions in conceptual spaces, which can move through a semantic space in time. The goal of this metaphor is to impose structure on the target domain and therefore support the explanation of its processes.

Table 1. Metaphorical projection from time-geographic to semantic-space elements

Time-geographic elements	**Semantic-space elements**
geographic space	semantic space
geographic distance (d_{geog})	semantic distance (d_{sem})
space-time path (ST-path)	semantic space-time path (SST-path)
space-time station (STS)	semantic space-time station (SSTS)
space-time prism (STP)	semantic space-time envelope (SSTE)
coupling constraint	semantic coupling constraint
authority constraint	contextual constraint
potential path space (PPS)	semantic potential path space (SPPS)

More specifically, individual time-geographic elements are being mapped to elements in the semantic space (Table 1, Figure 3). Geographic space is being mapped to *semantic space*, which can be thought of as a two- or three-dimensional attribute surface as used in information visualization [32, 33]. Both conceptual spaces and semantic spaces have a metric, which allows for measuring *semantic distances* d_{sem} between concepts and conceptual spaces [19]. Conceptual spaces (CS_1 and CS_2 in Figure 3) move along *semantic space-time paths* (SST-path), vertical paths thereby signifying stationary semantics, i.e., no conceptual change involving a change in dimensions but changes in dimension values are possible (see Section 3.3). Such stationarity corresponds to a *semantic space-time station* (SSTS). The *semantic space-time envelope* (SSTE) and *semantic potential path space* (SPPS) define through their boundaries, how far a conceptual space (including its concept regions) can deviate from a vertical path and still represent the same or similar semantics. Crossing the boundaries corresponds to conceptual change. It is important to note that these boundaries are often fuzzy and indeterminate [34]. The extent of the SSTE is a function of time depending on the changes in the semantic space as defined above.

The partial mapping from source to target domain includes two constraints. Coupling constraints are being mapped to *semantic coupling constraints*, which specify the interaction of conceptual spaces (and concepts) based on the coincidence of their

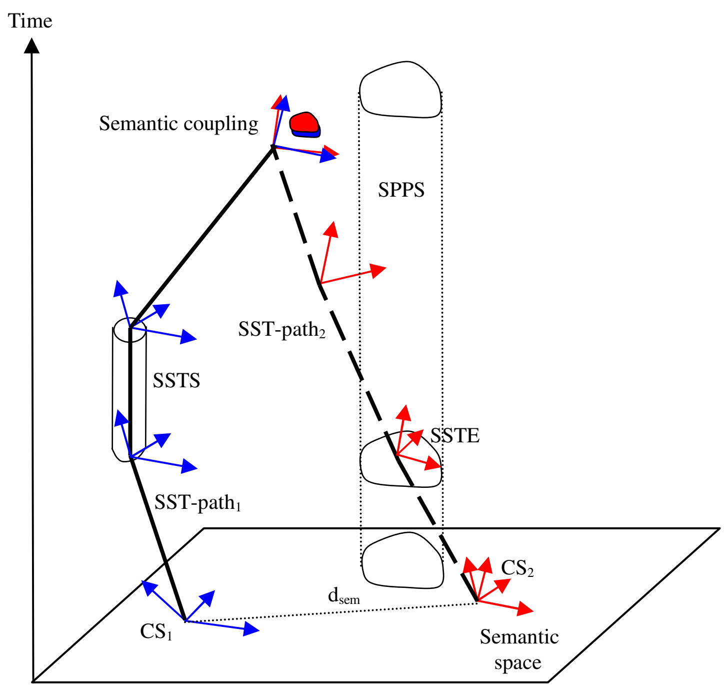

Fig. 3. Representation of moving conceptual spaces in a semantic space over time. For clarity reasons, the concept regions are only visualized once (during semantic coupling).

semantic space-time paths (i.e., semantic space-time bundling). Such coincidence signifies high (significant overlap of concept regions, see Figure 3) or even total conceptual similarity, e.g., when two different concepts merge into one over time, such as the abstract political concepts of Eastern and Western Germany. Authority constraints are being mapped to *contextual constraints*. Similar to fiat restrictions on activities in space and time, there exist legal definitions, such as traffic codes or data transfer standards, which create fiat conceptual boundaries. For example, the definition and meaning of terms, such as parcel or forest, depend on the legal system of the responsible administration—see also the discussion of institutional reality in [35]. The same symbol can therefore relate to different concepts represented by different dimensions or different regions in a conceptual space.

3.3 Within- and between-Conceptual-Space Changes

Our proposed mental model allows for representing conceptual change over time from two perspectives, namely (a) change of the geometrical structure of concepts as n-dimensional regions *within* one conceptual space and (b) changes *between* different conceptual spaces. Case (a) presumes that no change of quality dimensions has occurred in the conceptual space, therefore allowing only for movement of the concept region within this particular space—caused by a change in dimension values. One can then measure the semantic distance between a concept c at time t_i and the same concept at time t_{i+1}. Three strategies for calculating semantic similarity between conceptual regions, including overlapping concepts, have been demonstrated in [20] and can be applied here. These methods differ, in that for each vector of $c(t_i)$ one or several corresponding vectors of $c(t_{i+1})$ are identified.

Case (b) applies to mappings between conceptual spaces, leading to a change in quality dimensions. These mappings can either be *projections*, which reduce the complexity of the space by reducing its number of dimensions, or *transformations*, which involve a major change of quality dimensions, such as the addition of new dimensions. As shown in [36], projections (Equation 1) and transformations (Equation 2) can be expressed as partial mappings with *C, D* denoting conceptual spaces and *m, n* the number of quality dimensions. For projections, the semantics of the mapped quality dimensions must not change or can be mapped by rules.

$$(R_{proj}: C^m \rightarrow D^n) \text{ where } n < m \text{ and } C^m \cap D^n = D^n \quad (1)$$

$$(R_{trafo}: C^m \rightarrow D^n) \text{ where } (n \leq m \text{ and } C^m \cap D^n \neq D^n) \text{ or } (n > m) \quad (2)$$

4 Formal Model of Conceptual Change in Time

This section develops a computational mental model for representing conceptual change in time according to the presented spatio-temporal metaphor. We take an algebraic approach to formally specify the mappings of entities and operations at the level of conceptual spaces (which represent the conceptual regions). These specifications will be used in Section 5 to demonstrate the applicability of the formal model.

4.1 Algebraic Specifications

Our method of formalization uses algebraic specifications, which present a natural way of representing entities and processes. Algebraic specifications have proven useful for specifying data abstractions in spatial and temporal domains [25, 37-39]. Data abstractions are based on abstract data types, which are representation-independent formal definitions of all operations of a data type [40]. Entities are described in terms of their operations, depicting how they behave. Algebraic specifications written in an executable programming language can be tested as a prototype [41]. The tool chosen here is Hugs, a dialect of the purely functional language Haskell [42], which includes types, type classes, and algebraic axioms. Haskell provides higher-order capabilities and one of its major strengths is strong typing: every object has a particular type and the compiler checks that operations can only be applied to certain types.

4.2 Formal Model

A conceptual space is formally specified[3] as a data type, together with its attributes. Every conceptual space has an identifier `Id`, a `Position` in the semantic space at a

[3] The complete Hugs code including the test data for this paper is available at http://www.geog.ucsb.edu/~raubal/Downloads/CS.hs. Hugs interpreters can be downloaded freely from http://www.haskell.org.

given `Time`, and consists of a number of quality dimensions (list `[Dimension]`). Every `Dimension` has a `Name` and a range of values (`ValueRange`) with a given `Unit`, e.g., dimension *weight* with values between *0* and *250 kg*. Here, we define `Position` as a coordinate pair in a 2-dimensional semantic space and `Time` through discrete steps.

```
data ConceptualSpace = NewConceptualSpace Id Position
     Time [Dimension]
data Dimension = Dimension Name ValueRange Unit
```

We can now define a type class with common functions for conceptual spaces. These functions can be simple operations to observe properties, such as the current position of a conceptual space (`getConceptualSpacePosition`), but also more complex operations that specify the elements, processes, and constraints described in Section 3. The abstract type signatures are implementation-independent and can therefore be implemented for different types of conceptual spaces. Here, we inherit the class behavior to the data type `ConceptualSpace` as specified above.

```
class ConceptualSpaces cs where
   getConceptualSpacePosition :: cs -> Position

instance ConceptualSpaces ConceptualSpace where
   getConceptualSpacePosition
      (NewConceptualSpace id pos t ds) = pos
```

Conceptual change happens through movement of conceptual spaces along space-time paths in the semantic space (and through movement of conceptual regions within conceptual spaces). Conceptual spaces move to new positions only if there is a change in dimensions (`dsNew`), otherwise they are stationary. The `semanticDistance` function calculates either how far one conceptual space has moved in the semantic space during a particular time interval, or the distance between two different conceptual spaces (such as d_{sem} in Figure 3). It is currently implemented for 2-D Euclidean distance (`dist`) but different instances of the Minkowski metric can be used instead, depending on the types of dimensions and spaces [1]. A `SemanticSpaceTime-Path` is constructed by finding (filtering) all conceptual space instances for a particular `Id` and ordering them in a temporal sequence.

```
class ConceptualSpaces cs where
    moveConceptualSpace :: cs -> [Dimension] ->
         ConceptualSpace
    semanticDistance :: cs -> cs -> Distance
    constructSemanticSpaceTimePath :: Id -> [cs] ->
         SemanticSpaceTimePath

instance ConceptualSpaces ConceptualSpace where
    moveConceptualSpace (NewConceptualSpace id pos t
       ds) dsNew
       = if ds == dsNew
         then (NewConceptualSpace id pos newT ds)
         else (NewConceptualSpace id newPos newT dsNew)
    semanticDistance (NewConceptualSpace id pos t ds)
       (NewConceptualSpace id2 pos2 t2 ds2)
       = dist pos pos2
```

```
constructSemanticSpaceTimePath i cs
   = NewSemanticSpaceTimePath id css
     where
     id = i
     css = filter ((i== ).getConceptualSpaceId) cs
```

Semantic space-time stations are specified as special types of `SemanticSpaceTimePaths`—similar to the representation of space-time stations in [43]—, i.e., consisting of conceptual space instances with equal positions (but potential temporal gaps). The derivation of a `SemanticSpaceTimeStation` is based on the sorting function `sortConceptualSpaces`, which orders conceptual spaces according to their positions.

```
class SemanticSpaceTimePaths sstPath where
    constructSemanticSpaceTimeStation :: sstPath ->
       [ConceptualSpace]

instance SemanticSpaceTimePaths SemanticSpaceTimePath
    where
    constructSemanticSpaceTimeStation
       (NewSemanticSpaceTimePath id cs)
       = sortConceptualSpaces cs
```

The data type `SemanticSpaceTimeEnvelope` is defined by a `Center` (of type `Position`) and a `Boundary` for each time step. The projection of SSTE to semantic space results in a region (equivalent to the PPA from time geography), whose boundary delimits a semantic similarity area. Note that contrary to semantic space-time stations, semantic potential path spaces—which result from integration over a sequence of SSTE slices—cannot have gaps. One can now determine algorithmically, whether a conceptual space falls inside the boundary or not (which identifies conceptual change).

```
data SemanticSpaceTimeEnvelope =
     NewSemanticSpaceTimeEnvelope Center Time Boundary
```

Semantic coupling constraints are represented through the `semanticMeet` function. It determines whether two instances of conceptual spaces interact at a given time step. This definition leaves room for integrating semantic uncertainty by specifying a threshold for the semantic distance (`epsilon`), within which the conceptual spaces are still considered to be interacting, see also [44]. Contextual constraints are fiat boundaries in the semantic space and can therefore be represented by the `Boundary` type.

```
class ConceptualSpaces cs where
    semanticMeet :: cs -> cs -> Bool

instance ConceptualSpaces ConceptualSpace where
    semanticMeet cs1 cs2
       = (getConceptualSpaceTime cs1 ==
       getConceptualSpaceTime cs2)
       && (semanticDistance cs1 cs2 <= epsilon)
```

5 Application: Geospatial Concept Change in Time

The formal model in the previous section provides executable specifications of the represented elements and processes for conceptual change based on the geometrical framework of conceptual spaces. In order to demonstrate the model with respect to analyzing the change of conceptual structures in time, we apply it to the use case of representing the concept of ‘landmark’ within the particular scenario of wayfinding in a city [45], where façades of buildings are often used as landmarks. Geospatial concepts, such as lake, mountain, geologic region, street, or landmark, differ in many qualitative ways from other concepts, due to their spatio-temporal nature [46, 47]. Their structure in terms of represented meaning changes for individual persons over time and may also differ between cultures, e.g., classifications of landscapes [48].

In the following, the change of a person’s conceptual structure of ‘landmark’ (in terms of façade as described above) over time is represented with respect to the change of quality dimensions in a semantic space. Based on previous work, we specify the dimensions façade area `fa` (square meters), shape deviation `sd` (deviation from minimum bounding rectangle in percent), color `co` (three RGB values), cultural importance `ci` (ordinal scale of 1 to 5), and visibility `vi` (square meters) [19, 45].

```
fa = (Dimension "area" (100,1200) "sqm")
sd = (Dimension "shape" (0,100) "%")
co = (Dimension "color" (0,255) "RGB")
ci = (Dimension "cultural" (1,5) "importance")
vi = (Dimension "visibility" (0,10000) "sqm")
```

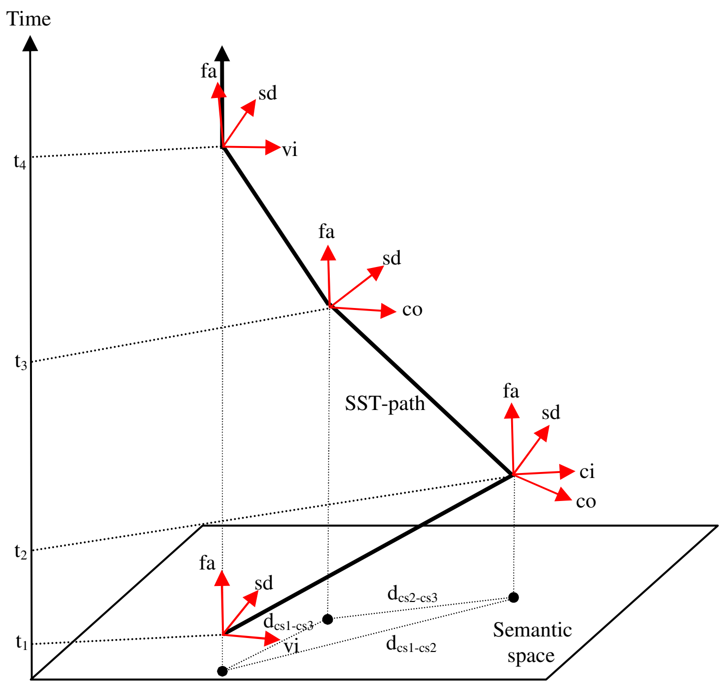

Fig. 4. Change of a person’s conceptual structure of ‘landmark’ over time

Four time steps are considered, which results in four instances of the conceptual space[4]. In this scenario, the person's 'landmark' concept comprises three quality dimensions at time t_1 (`cs1`). Through experience and over the years, the person has acquired a sense of cultural importance of buildings (`cs2`)—a building may be famous for its architectural style, therefore being a landmark—, adding this new dimension and also the significance of color. Next, for the reason of variation in the person's interests, cultural importance vanishes again (`cs3`). Over time, due to physiological changes resulting in color blindness, the person's concept structure changes back to the original one, eliminating color and again including visibility. Figure 4 visualizes these conceptual changes over time.

```
cs1 = NewConceptualSpace 1 (3,1) 1 [fa,sd,vi]
cs2 = NewConceptualSpace 1 (6,3) 2 [fa,sd,ci,co]
cs3 = NewConceptualSpace 1 (4,2) 3 [fa,sd,co]
cs4 = NewConceptualSpace 1 (3,1) 4 [fa,sd,vi]
```

The formal specifications can now be used to query the temporal conceptual representation in order to find conceptual changes and when they happened, and what semantics is represented by a particular conceptual structure at a specific time. We can infer that the semantic change from `cs1` at time `1` to `cs2` at time `2` (transformation with two new dimensions) is larger than the change from `cs1` at time `1` to `cs3` at time `3` (transformation with one new dimension) by calculating the respective semantic distances ($d_{cs1\text{-}cs2}$ and $d_{cs1\text{-}cs3}$ in Figure 4). The change resulting from the move between time `2` and `3` ($d_{cs2\text{-}cs3}$) is due to a projection, involving a reduction to three dimensions. Similarity is thereby a decaying function of semantic distance, which depends on the semantic space. The interpretation of semantic distance is domain-dependent and may be determined through human participants tests [49].

```
semanticDistance cs1 cs2
3.605551
semanticDistance cs1 cs3
1.414214
semanticDistance cs2 cs3
2.236068
```

We can further construct the semantic space-time path for the conceptual space under investigation from the set of all available conceptual space instances (`allCs`). The result (presented below is only the very beginning for space reasons) is a list of the four conceptual space instances with `Id=1` in a temporal sequence. This SST-path is visualized in Figure 4.

```
constructSemanticSpaceTimePath 1 allCs
[NewSemanticSpaceTimePath 1 [NewConceptualSpace 1 …]
```

Applying the `constructSemanticSpaceTimeStation` function to the SST-path derives all conceptual space instances with equal positions but potentially temporal gaps, such as `cs1` and `cs4`.

[4] The quantitative values for the positions of conceptual spaces in the semantic space are for demonstration purposes. Their determination, such as through similarity ratings from human participants tests, is left for future work.

```
constructSemanticSpaceTimeStation
   (constructSemanticSpaceTimePath 1 allCs)
[NewConceptualSpace 1 (3.0,1.0) 1 [Dimension "area"
(100.0,1200.0) "sqm",Dimension "shape" (0.0,100.0)
"%",Dimension "color" (0.0,255.0) "RGB"],
NewConceptualSpace 1 (3.0,1.0) 4 [Dimension "area"
(100.0,1200.0) "sqm",Dimension "shape" (0.0,100.0)
"%",Dimension "color" (0.0,255.0) "RGB"]]
```

6 Conclusions and Future Work

This paper presented a novel computational model to represent conceptual change over time. The model is based on a spatio-temporal metaphor, utilizing elements from time geography and conceptual spaces. Conceptual change is represented through movement of conceptual spaces along space-time paths in a semantic space. We developed executable algebraic specifications for the mapped entities, relations, and operations, which allowed demonstrating the model through an application to a geospatial conceptual structure. This application showed the potential of the formal representation for analyzing the dynamic nature of concepts and their changes in time.

The presented work suggests several directions for future research:

- The formal model needs to be extended to represent conceptual regions *within* the conceptual spaces. This will allow the application of semantic similarity measures, such as the ones proposed in [20], to determine semantic distances between individual concepts anchored within their corresponding conceptual spaces.
- The quantification of conceptual change depends on the representation of the semantic space, which we have modeled as a two-dimensional attribute surface. More research in cognitive science and information science is required to establish cognitively plausible, semantic surface representations (similar to those developed in the area of information visualization) for different domains that can be used within our proposed model. This will also determine the distance and direction when moving a conceptual space due to a change in its quality dimensions.
- Conceptual regions often do not have crisp boundaries therefore their representation must take aspects of uncertainty into account. Uncertainty also propagates when applying operations such as intersection to concept regions. Future work must address these issues based on the time-geographic uncertainty problems identified in [43].
- The semantic space is a similarity space, i.e., distance represents similarity between concepts. This leads to the question whether disparate concepts, such as roundness and speed, can be compared at all? A possible solution is to make concepts comparable only when they are within a certain threshold distance: if this is exceeded, then the similarity is zero. Another way is to specifically include infinite distance. It is essential to account for the given context in which concepts are compared. The context can be represented through different dimension weights.
- The formal specifications serve as the basis for implementing a concept query language, which can be tested in different application domains. This will help understanding various concept dynamics, more specifically, the characterization and prediction of conceptual change through time.

- In this work we utilized Gärdenfors' [1] notion of conceptual spaces as a geometric way of representing information at the conceptual level. Different views on the nature of conceptual representations in the human cognitive system exist, such as the ideas of mental images [50] or schematic perceptual images extracted from modes of experience [8]. Could such images be represented in or combined with conceptual spaces? Would such combination be similar to a cognitive collage [51]? Human participants tests may help assess the validity of geometrical representations of concepts and point to potential limitations of conceptual spaces as a representational model.

Acknowledgments

The comments from Carsten Keßler and three anonymous reviewers provided useful suggestions to improve the content of the paper.

Bibliography

1. Gärdenfors, P.: Conceptual Spaces - The Geometry of Thought. MIT Press, Cambridge (2000)
2. Hägerstrand, T.: What about people in regional science? Papers of the Regional Science Association 24, 7–21 (1970)
3. Brodaric, B., Gahegan, M.: Distinguishing Instances and Evidence of Geographical Concepts for Geospatial Database Design. In: Egenhofer, M., Mark, D. (eds.) Geographic Information Science - Second International Conference, GIScience 2002, Boulder, CO, USA, September 2002, pp. 22–37. Springer, Berlin (2002)
4. Berners-Lee, T., Hendler, J., Lassila, O.: The Semantic Web. In: Scientific American, pp. 34–43 (2001)
5. Gruber, T.: A Translation Approach to Portable Ontology Specifications. Knowledge Acquisition 5(2), 199–220 (1993)
6. Laurence, S., Margolis, E.: Concepts and Cognitive Science. In: Margolis, E., Laurence, S. (eds.) Concepts - Core Readings, pp. 3–81. MIT Press, Cambridge (1999)
7. Smith, E.: Concepts and induction. In: Posner, M. (ed.) Foundations of cognitive science, pp. 501–526. MIT Press, Cambridge (1989)
8. Barsalou, L., Yeh, W., Luka, B., Olseth, K., Mix, K., Wu, L.: Concepts and meaning. In: Beals, K., et al. (eds.) Parasession on conceptual representations, pp. 23–61. University of Chicago, Chicago Linguistics Society (1993)
9. Goldstone, R., Kersten, A.: Concepts and Categorization. In: Healy, A., Proctor, R. (eds.) Comprehensive handbook of psychology, pp. 599–621 (2003)
10. Piaget, J., Inhelder, B.: The Child's Conception of Space. Norton, New York (1967)
11. Neisser, U.: Cognition and Reality - Principles and Implications of Cognitive Psychology. Freeman, New York (1976)
12. Lakoff, G.: Cognitive Semantics, in Meaning and Mental Representations. In: Eco, U., Santambrogio, M., Violi, P. (eds.), pp. 119–154. Indiana University Press, Bloomington (1988)
13. Green, R.: Internally-Structured Conceptual Models in Cognitive Semantics. In: Green, R., Bean, C., Myaeng, S. (eds.) The Semantics of Relationships - An Interdisciplinary Perspective, pp. 73–89. Kluwer, Dordrecht (2002)

14. Kuhn, W., Raubal, M., Gärdenfors, P.: Cognitive Semantics and Spatio-Temporal Ontologies. Spatial Cognition and Computation 7(1), 3–12 (2007)
15. Ogden, C., Richards, I.: The Meaning of Meaning: A Study of the Influence of Language Upon Thought and of the Science of Symbolism. Routledge & Kegan Paul, London (1923)
16. Barsalou, L.: Situated simulation in the human conceptual system. Language and Cognitive Processes 5(6), 513–562 (2003)
17. Sowa, J.: Categorization in Cognitive Computer Science. In: Cohen, H., Lefebvre, C. (eds.) Handbook of Categorization in Cognitive Science, pp. 141–163. Elsevier, Amsterdam (2006)
18. Gärdenfors, P.: Representing actions and functional properties in conceptual spaces. In: Ziemke, T., Zlatev, J., Frank, R. (eds.) Body, Language and Mind, pp. 167–195. Mouton de Gruyter, Berlin (2007)
19. Raubal, M.: Formalizing Conceptual Spaces, in Formal Ontology in Information Systems. In: Varzi, A., Vieu, L. (eds.) Proceedings of the Third International Conference (FOIS 2004), pp. 153–164. IOS Press, Amsterdam (2004)
20. Schwering, A., Raubal, M.: Measuring Semantic Similarity between Geospatial Conceptual Regions. In: Rodriguez, A., et al. (eds.) GeoSpatial Semantics - First International Conference, GeoS 2005, Mexico City, Mexico, November 2005, pp. 90–106. Springer, Berlin (2005)
21. Devore, J., Peck, R.: Statistics - The Exploration and Analysis of Data, 4th edn. Duxbury, Pacific Grove (2001)
22. Lenntorp, B.: Paths in Space-Time Environments: A Time-Geographic Study of the Movement Possibilities of Individuals. Lund Studies in Geography, Series B (44) (1976)
23. Miller, H.: Modeling accessibility using space-time prism concepts within geographical information systems. International Journal of Geographical Information Systems 5(3), 287–301 (1991)
24. Palmer, S.: Fundamental aspects of cognitive representation. In: Rosch, E., Lloyd, B. (eds.) Cognition and categorization, pp. 259–303. Lawrence Erlbaum, Hillsdale (1978)
25. Frank, A.: Spatial Communication with Maps: Defining the Correctness of Maps Using a Multi-Agent Simulation. In: Freksa, C., et al. (eds.) Spatial Cognition II - Integrating Abstract Theories, Empirical Studies, Formal Methods, and Practical Applications, pp. 80–99. Springer, Berlin (2000)
26. Frank, A.: Pragmatic Information Content: How to Measure the Information in a Route Description. In: Duckham, M., Goodchild, M., Worboys, M. (eds.) Foundations of Geographic Information Science, pp. 47–68. Taylor & Francis, London (2003)
27. Lakoff, G., Johnson, M.: Metaphors We Live By. University of Chicago Press, Chicago (1980)
28. Kuipers, B.: The 'Map in the Head' Metaphor. Environment and Behaviour 14(2), 202–220 (1982)
29. Kuhn, W.: Metaphors Create Theories for Users. In: Frank, A.U., Campari, I. (eds.) Spatial Information Theory: Theoretical Basis for GIS, pp. 366–376. Springer, Berlin (1993)
30. Kuhn, W., Blumenthal, B.: Spatialization: Spatial Metaphors for User Interfaces. Geoinfo-Series, vol. 8. Department of Geoinformation, Technical University Vienna, Vienna (1996)
31. Lakoff, G.: Women, Fire, and Dangerous Things: What Categories Reveal About the Mind. The University of Chicago Press, Chicago (1987)
32. Skupin, A.: Where do you want to go today [in attribute space]? In: Miller, H. (ed.) Societies and Cities in the Age of Instant Access, pp. 133–149. Springer, Dordrecht (2007)

33. Skupin, A., Fabrikant, S.: Spatialization Methods: A Cartographic Research Agenda for Non-Geographic Information Visualization. Cartography and Geographic Information Science 30(2), 95–119 (2003)
34. Burrough, P., Frank, A., Masser, I., Salgé, F.: Geographic Objects with Indeterminate Boundaries. GISDATA Series. Taylor & Francis, London (1996)
35. Frank, A.: Ontology for spatio-temporal Databases. In: Koubarakis, M., et al. (eds.) Spatio-temporal Databases: The Chorochronos Approach, pp. 9–77. Springer, Berlin (2003)
36. Raubal, M.: Mappings For Cognitive Semantic Interoperability. In: Toppen, F., Painho, M. (eds.) AGILE 2005 - 8th Conference on Geographic Information Science, pp. 291–296. Instituto Geografico Portugues (IGP), Lisboa (2005)
37. Winter, S., Nittel, S.: Formal information modelling for standardisation in the spatial domain. International Journal of Geographical Information Science, 2003 17(8), 721–742 (2003)
38. Raubal, M., Kuhn, W.: Ontology-Based Task Simulation. Spatial Cognition and Computation 4(1), 15–37 (2004)
39. Krieg-Brückner, B., Shi, H.: Orientation Calculi and Route Graphs: Towards Semantic Representations for Route Descriptions. In: Raubal, M., et al. (eds.) Geographic Information Science, 4th International Conference GIScience 2006, Muenster, Germany, pp. 234–250. Springer, Berlin (2006)
40. Guttag, J., Horowitz, E., Musser, D.: The Design of Data Type Specifications. In: Yeh, R. (ed.) Current Trends in Programming Methodology, pp. 60–79. Prentice-Hall, Englewood Cliffs (1978)
41. Frank, A., Kuhn, W.: Specifying Open GIS with Functional Languages. In: Egenhofer, M., Herring, J. (eds.) Advances in Spatial Databases (SSD 1995), pp. 184–195. Springer, Portland (1995)
42. Hudak, P.: The Haskell School of Expression: Learning Functional Programming through Multimedia. Cambridge University Press, New York (2000)
43. Miller, H.: A Measurement Theory for Time Geography. Geographical Analysis 37(1), 17–45 (2005)
44. Ahlqvist, O.: A Parameterized Representation of Uncertain Conceptual Spaces. Transactions in GIS 8(4), 493–514 (2004)
45. Nothegger, C., Winter, S., Raubal, M.: Selection of Salient Features for Route Directions. Spatial Cognition and Computation 4(2), 113–136 (2004)
46. Smith, B., Mark, D.: Geographical categories: an ontological investigation. International Journal of Geographical Information Science 15(7), 591–612 (2001)
47. Brodaric, B., Gahegan, M.: Experiments to Examine the Situated Nature of Geoscientific Concepts. Spatial Cognition and Computation 7(1), 61–95 (2007)
48. Mark, D., Turk, A., Stea, D.: Progress on Yindjibarndi Ethnophysiography. In: Winter, S., et al. (eds.) Spatial Information Theory, 8th International Conference COSIT 2007, Melbourne, Australia, pp. 1–19. Springer, Berlin (2007)
49. Hahn, U., Chater, N.: Understanding Similarity: A Joint Project for Psychology, Case-Based Reasoning, and Law. Artificial Intelligence Review 12, 393–427 (1998)
50. Kosslyn, S.: Image and brain - The resolution of the imagery debate. MIT Press, Cambridge (1994)
51. Tversky, B.: Cognitive Maps, Cognitive Collages, and Spatial Mental Model. In: Frank, A., Campari, I. (eds.) Spatial Information Theory: Theoretical Basis for GIS, pp. 14–24. Springer, Berlin (1993)