Formal Specification of Image Schemata – A Step to Interoperability in Geographic Information Systems

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Abstract

The formal specification of spatial objects and spatial relations is at the core of geographic data exchange and interoperability for geographic information systems (GIS). It is necessary that the representation of such objects and relations comes close to how people use them in their everyday lives, i.e., that these specifications are built upon elements of human spatial cognition. Image schemata have been suggested as highly abstract and structured mental patterns to capture spatial and similar physical as well as metaphorical relations between objects in the experiential world. We assume that image-schematic details for large-scale (geographic) space are potentially different from image-schematic details for small-scale (table-top) space. This paper reviews methods for the formal description of spatial relations, integrates them in a categorical view, and applies the methods arrived at to formally specify image schemata for large-scale (LOCATION, PATH, REGION, and BOUNDARY) as well as small-scale (CONTAINER, SURFACE, and LINK) space. These specifications should provide a foundation for further research on formalizing elements of human spatial cognition for interoperability in GIS.

1 Introduction

Exchange of data between GIS and interoperability of different vendors' GIS software are topics of enormous practical interest (Buehler and McKee 1996). Unambiguous definitions are at the core of any effort to achieve the necessary standardization that allows data exchange and cooperation of different GIS.

Standardization of technical terms and the fundamental concepts necessary to make computers interact is mostly achieved or can be achieved with current tools. The abstract behavior of computerized systems can be specified in a formal language and it requires then the checking of the compliance of the target computer system—which is by definition also a formal system—with the abstract formal system. This problem is not specific to GIS but general for all computer system standardization. The difficulties are of a practical nature and related to the lack of formal definition of most current computer languages, commercial interests in maintaining incompatible systems, and the rapid development compounded with legacy systems.

The economically important and scientifically challenging question is to describe the meaning of GIS data in terms of the real world, i.e., the so-called "semantics problem." What does it mean that "P 271" is a point, "343a" a land parcel, that building "A1" is on parcel "343a", A-town is on the B-river etc., and how is this meaning communicated between systems. The naive assumption that a "rose is a rose is a rose" (Gertrude Stein) is obviously not correct: the definitions of simple geographic properties differ from country to country, despite corresponding names (Chevallier 1981, Mark 1993, Kuhn 1994).

Image schemata describe high-level, abstract structures of common situations, most of them expressing spatial relations (Johnson 1987). Image schemata (Johnson 1987, Lakoff 1987) are the fundamental experiential elements from which spatial meaning is constructed, but so far image schemata have mostly resisted formal descriptions. This paper formalizes a number of image schemata important in the geographic context (LOCATION, PATH, REGION, and BOUNDARY) and in table-top space (CONTAINER, SURFACE, and LINK). This investigation is, therefore, part of the quest for naive or commonsense physics (Hayes 1978, Hayes 1985, Hobbs and Moore 1985) and in particular for "Naive Geography" (Egenhofer and Mark 1995).

The next section argues why the formalization of spatial relations in geographic space is crucial for further advances in the standardization and interoperability of GIS. Despite large strides in some small specialized areas—in particular topological relations—not much progress has been made in general. The program to formalize spatial image schemata as conceived by humans has not been completed yet. In Section 3 the specification of image schemata is discussed and Section 4 describes methods to formalize image schemata. Section 5 gives a comprehensive method—built upon linguistics—to discover and formally describe image schemata. Section 6 explains the geographic-space-image-schemata (i.e., LOCATION, PATH, REGION, and BOUNDARY) considered and presents their formalizations. Section 7 explains the table-top image schemata (i.e., CONTAINER, SURFACE, and LINK) considered and presents their formalizations. Section 8 presents conclusions, discusses open questions, and suggests directions for further research.

2 Formalizing Spatial Meaning

The spatial domain—in which GIS facts are situated—is fundamental for human living and one of the major sources for human experience (Barrow 1992). Human language exploits the communality of spatial experience among people and uses spatial situations metaphorically to structure purely abstract situations in order to communicate them (Lakoff and Johnson 1980, Johnson 1987). The formalization of spatial relations has, therefore, been an active area of research at least since 1989 (Mark *et al.* 1995).

Topological relations between simply connected regions were treated in (Egenhofer 1989) and extensive work has followed from this (Egenhofer 1994). Metric relations between point-like objects, especially cardinal directions (Frank 1991b, Frank 1991a, Freksa 1991, Hernández 1991) and approximate distances (Frank 1992, Hernández *et al.* 1995, Frank 1996b) were discussed. Other efforts dealt with orderings among configurations of points (Schlieder 1995) and formal descriptions of terrain and relations in terrain (Frank *et al.* 1986), but formal methods were also used to formally describe the working of administrative systems (e.g., cadastre (Frank 1996a)). Linguists have made systematic efforts to clarify the meaning of spatial prepositions (Herskovits 1986, Lakoff 1987). However, it remains an open question how to combine these interesting results within a uniform system and to apply them systematically to other examples.

The specification of spatial relations is of great practical interest to define spatial relations in spatial query languages unambiguously; the current plethora of proposals for spatial relations to complete database query languages is useless unless the relations are formally specified (which is the case for the standard relations in SQL) (Egenhofer 1992). The formal properties are the base for query optimization. Image schemata are considered good candidates as a foundation for the formal definition of spatial relations. Kuhn has pointed out the importance of image schemata as a tool to build ”natural” (i.e., cognitively sound) user interfaces for GIS (Kuhn and Frank 1991, Kuhn 1993).

3 Specification of Image Schemata

Johnson (1987) proposes that people use recurring, imaginative patterns—so-called *image schemata*—to comprehend and structure their experiences while moving through and interacting with their environment. Image schemata are supposed to be pervasive, well-defined, and full of sufficient internal structure to constrain people’s understanding and reasoning. They are more abstract than mental pictures and less abstract than logical structures because they are constantly operating in people’s minds while people are experiencing the world (Kuhn and Frank 1991). An image schema can, therefore, be seen as a generic, maybe universal, and abstract structure that helps people establish a connection between different experiences that have this same recurring structure in common. Table 1 gives the partial list of Johnson’s (1987 p.126) image schemata.

Container	Balance	Compulsion
Blockage	Counterforce	Restraint Removal
Enablement	Attraction	Mass-Count
Path	**Link**	**Center-Periphery**
Cycle	Near-Far	Scale
Part-Whole	Merging	Splitting
Full-Empty	Matching	Superimposition
Iteration	Contact	Process
Surface	Object	Collection

Table 1: Partial list of image schemata (Johnson 1987 p.126)—in bold the spatial image schemata treated here.

Although the theory of image schemata has not been universally accepted, one can find empirical evidence for the existence and relevance of image-schemata-like phenomena in existing literature. In a pilot study, Freundschuh and Sharma (1996) assessed the geographical content of children's narratives and investigated the relationship between locatives and spatial image schemata. One of their results was that books for different age levels utilized a standard set of locatives, suggesting the possibility to express most spatial relationships (i.e., spatial image schemata) with few locative terms. They also found indications that some image schemata (e.g., the CONTAINER schema) are more fundamental than others, demonstrating a possible developmental sequence in the building and comprehension of spatial image schemata. In a case study about wayfinding in airports Raubal *et al.* (1997) showed that people use image schemata and metaphorical projections of image schemata to structure their wayfinding tasks. Subsequent work (Raubal and Egenhofer 1998) presented a computational method to compare the complexity of wayfinding tasks in built environments. Image schemata were used to determine the critical elements of a wayfinding model.

3.1 Previous Formal Description of Image Schemata

Despite efforts, success in specifying spatial image schemata has been limited. An early paper (Kuhn and Frank 1991) gave algebraic definitions for the CONTAINER (”in”) and SURFACE (”on”) schemata for a discussion of user interface design. At the level of detail and for the purpose of the paper, the two specifications were isomorphic. A recent effort by Rodríguez and Egenhofer (1997) introduced more operations and differentiated the CONTAINER schema from the SURFACE schema for small-scale space, using operations such as *remove*, *jerk*, and *has-_contact*, and compared the application to objects in small-scale and large-scale (geographic) space. In (Raubal *et al.* 1997) image schemata were represented in the form of predicates in which the predicate name referred to the image schema and the argument(s) referred to the object(s) involved in the image schema (see also Raubal 1997).

In a recent paper (Frank 1998) formal descriptions for the small-scale-space-image-schemata CONTAINER, SURFACE, and LINK were given (corresponding to the German prepositions “in”, “an”, and “auf”) and some of the methodological difficulties reviewed. The large-scale-space-image-schemata LOCATION, PATH, REGION, and BOUNDARY were treated in (Frank and Raubal 1998).

3.2 Definition of the Concept of an Image Schema

The concept of image schemata is not well-defined in the cognitive and linguistic literature (Lakoff and Johnson 1980, Johnson 1987, Lakoff 1987). Researchers in the past have used a working definition that implied that image schemata describe spatial (and similar physical) relations between objects. Most have concentrated on spatial prepositions like “in”, “on”, etc. and assumed that these relate directly to the image schemata (Freundschuh and Sharma 1996, Raubal *et al.* 1997, Raubal and Egenhofer 1998).

Image schemata are seen as fundamental and independent of the type of space and spatial experience. But a single schema can appear in multiple, closely related situations. For example, “in” is used for a bowl of fruit (“Der Apfel ist in der Schale.”—“The apple is in the fruit bowl.”), but also for closed containers (“Das Geld ist im Beutel.”—“The money is in the purse.”). “Prototype effects” as described by Rosch (1973a, 1973b, 1978) also seem to apply. For example, a different level of detail can be selected to describe the same image schema.

3.3 Language Dependence of Expressing Image Schemata

It is possible that image schemata provide universal structural building blocks, but different languages may combine the building blocks differently; also, people with different native languages may use another set of language-specific features of spatial encoding (Bowerman 1996) and therefore linguistically express particular image schemata differently. The obvious differences between languages are one important point in the cultural distinction that limits the use of GIS (Campari and Frank 1995) and the problem is further aggravated by regional differences within a language.

4 Methods to Formalize Image Schemata

4.1 Predicate Calculus

Lakoff (1987) gives a definition of the CONTAINER schema using predicate calculus. In theory, predicate calculus has all the expressive power necessary, but it is practically limited by the frame problem, which makes succinct definition for changes impossible (Hayes 1977, McCarthy 1985). McCarthy (1980, 1986) proposed situation calculus with circumscription as an extension of the logical theory to overcome this limitation.

4.2 Relations Calculus

The behavior of topological relations (Egenhofer 1994, Papadias and Sellis 1994), but also cardinal directions and approximate distances (Hernández *et al.* 1995, Freksa 1991, Frank 1992, Frank 1996b) can be analyzed using the relations calculus (Schroeder 1895, Tarski 1941, Maddux 1991). Properties of relations are described as the outcome of the combination (the “;” operator) of two relations. The description abstracts away the individuals related (in comparison to the predicate calculus) and gives a simple algebra over relations. This leads to succinct and easy-to-read tables, as long as the combination of only a few relations is considered.

a (R;S) c = aRb and bSc

for example: North;NorthEast = {North or NorthEast}

meet;inside = {inside, covered, overlap}

4.3 Functions

Functions are more appropriate to capture the semantics of image schemata with respect to operations. Relation composition is replaced by function composition (the "." operator). In order to use this notation flexibly, a "curried" form (i.e., replacing structured arguments by a sequence of simple ones) of function writing must be used (Bird and Wadler 1988, Bird and Moor 1997).

$$f \, . \, g\,(x) = f\,(g\,(x)).$$

Function composition can be described by tables as well, but these grow even faster than relation composition tables. Axiomatic descriptions as algebras are more compact but more difficult to read.

4.4 Model Based

A model of the scene is constructed and used for reasoning (there is some evidence that this is also one of the methods humans apply (Knauff *et al.* 1995)). A fundamental set of operations to construct any possible state of the model and a sufficient number of "observe" operations to differentiate any of these states are provided. In addition, more complex operations can be constructed using the given operations.

The simplest model is to use the constructors of the scene directly and to represent each scene as the sequence of constructors which created it (Rodríguez 1997). This gives a (possibly executable) model for functional or relation oriented description.

Such models can be ontological—modeling some subset of the existing world—or they can be epistemological—modeling exclusively the human conceptualization of the world. More than one epistemological view can follow from an ontological model.

4.5 Tools Used

Formal specifications written and checked only by human minds must be regarded with great skepticism: humans are not particularly apt in finding errors in formal descriptions. For effective work, formal (computerized) tools must be used. Two types have been used: Logic-based languages (e.g., Prolog (Clocksin and Mellish 1981)), used for the definition of spatial terminology (Frank *et al.* 1986) and for spatial relations calculus (Egenhofer 1989). Logic-based systems must use "extralogical" operations when change is considered (*assert* and *retract* in Prolog). Recently, functional languages (Bird and Wadler 1988) have been advocated (Frank 1994, Kuhn and Frank 1997), especially Haskell (Peterson *et al.* 1997) and Gofer (Jones 1991, Jones 1994). Allegories (a special kind of categories) provide the theoretical structure to unify the two approaches (Bird and Moor 1997).

5 A Linguistic Method to Discover and Describe Image Schemata

Language has been used for studying any aspect of cognition, because the method is both convenient and the social sciences have had a long standing tradition with a rich set of linguistic methods and theories. It has been used to study spatial cognition, because the “grammar and syntax of a language, its lexicon and etymology, its semantics, pragmatics, and use all can provide valuable information and insights about human spatial cognition (Mark 1997).” Mark and Frank (1996) showed how image schemata can be deduced from natural-language expressions describing geographic situations. The image schema that has been in the speaker’s mind while making a statement can be inferred from the preposition (e.g., in, on, under) used (Mark 1989). The same approach was used by Freundschuh and Sharma (1996), Raubal *et al.* (1997), Raubal and Egenhofer (1998), and Frank (1998). A number of restrictions and assumptions are necessary to make progress with this line of investigation:

5.1 Operational Definition of Image Schemata

As an operational definition of image schemata we consider spatial-situations-image-schemata if they can be used as a source domain for metaphorical transfer to some target domain; this demonstrates that a commonly understood structural content, that is independent of the specific situation, exists.

5.2 Assumption of Polysemy

A single word may have multiple meanings (e.g., the English word ”spring” can be the verb ”to jump”, a season, a source, etc.). We assume that polysemy helps to initially separate what are potentially different meanings of a word for formalization. If the meanings are the same after formal description is achieved, the assumed polysemy can be dropped. In particular, we assume here that spatial prepositions are polysemous when used for different types of spaces, i.e., “in” for small-scale (table-top) space as investigated in (Rodríguez and Egenhofer 1997, Frank 1998) and “in” for geographic space are assumed to be two homonyms.

5.3 Exclusion of Partial Spatial Relations

Spatial relations may be partial: a pen may be partially on a sheet of paper, a city partially in one, partially in another state or country (e.g., Niagara Falls is a city both in Canada and the U.S.A.). At the present time such situations are excluded from consideration and their analysis is postponed. Ongoing work by Egenhofer (Rashid *et al.* 1998) to differentiate situations with the same topology by metric measures characterizing the degree of overlap etc. may answer these questions.

5.4 Restriction to a Single Level of Detail and Abstraction

The level of abstraction differs depending on the requirements of the situation (Timpf *et al.* 1992, Voisard and Schweppe 1994, Voisard and Schweppe 1997). These different levels of detail play an especially important part in geographic space and make the specification of image schemata difficult. Level of detail may be spatial subdivision, may be the consideration of additional rules, or may be the subdivision of categories into subcategories (Jordan *et al.* 1998, Giunchiglia and Walsh 1992). All these effects are excluded from this investigation.

5.5 Different image-schematic details for geographic and table-top spaces

We assume that image-schematic details for geographic space are separate from image-schematic details for small-scale space (Montello 1993, Couclelis 1992). Some of Johnson's (1987) suggested image schemata use terminology from geographic space (e.g., PATH), others suggest that the same image schema (e.g., SURFACE) is used for different types of spaces. If the same terminology is used, we assume here—for methodological reasons—polysemy (i.e., the same word is used with different meanings).

5.6 Concentration on a Single Language and Epistemology

The examples given here are in German (with English translations) as this is the authors' native language; the results can be compared with the English language situation and some differences observed (Herskovits 1986, Montello 1995). The language examples are the driving force here and the concentration is on the epistemology.

5.7 Method of Formalization

The two cases of formalization (i.e., geographic and table-top space) show a separation of the formalization of image schemata in two steps: First, static relations between objects in the scene are discussed—this could be achieved with relation algebra in all cases where only a few relations are concerned—and then the changes in relations with respect to operations that change the scene, which must necessarily be formalized with algebraic methods.

Image schemata are richly structured mental patterns that describe not only static situations but also classify situations with respect to the possible execution of operations (e.g., movement). Operations have preconditions that are expressed as relations, and here image schemata link operations and relations. Image schemata are, therefore, closely linked to the concept of affordances (Norman 1988).

The first case study—geographic space—is rich in derived spatial relations (subsections 6.2 to 6.4) and only when we consider the movement of persons in the landscape (subsection 6.5) preconditions and changes in the scene—of which the person is part—must be discussed. The relations among geographic objects are static and can, therefore, be formalized with predicate calculus. For each given relation, a converse relation exists. Relations are written in a prefix notation (similar to a predicate). Path (a, b) means there exists a path from a to b. This world is closed in the logical sense (Reiter 1984): everything is known about the scene and what is not specified can be assumed to be false. In particular, there are no unknown objects, all objects have different names, and all relations are known or inferred from the image schemata.

In the second case study—table-top space—we concentrate on the affordability of movement. Again, for each relation we have a converse *(a (conv Rel) b = b Rel a).* The spatial relations and their converses are interpreted as Boolean functions *fRel (a, b) -> Bool*, or functions which return for an object the relatum *fRel (a) -> b = a Rel b.* We say that an object *participates* in a spatial relation *Rel* if the corresponding *fRel* returns an object (this is equivalent to *Exist b: a Rel b*).

Both large-scale space and table-top space are relevant for GIS. GIS contain information primarily about large-scale space. In large-scale space objects are larger than people and therefore, can not be moved. People have to navigate through large-scale space, such as landscapes, cities, and houses, in order to learn about it. Considering geographic space from a less restricted perspective, objects may move (e.g., landslides or blockage of a path by obstacles) and then rules similar to those found in small-scale space apply. GIS-representations of large-scale space are mappings to small-scale space, because representations are smaller than people (e.g., houses on a computer screen) and can be manipulated. It has also been argued that large-scale space can be imagined only via reduction to small-scale space. Image schemata are a means to establish a connection between different experiences that have the same recurring structure in common. For example, the CONTAINER schema as used in small-scale space (chapter 7) is similar to the REGION schema as used in large-scale space (chapter 6).

6 Formal Specifications of Image Schemata for Large-Scale Space

The particular sets of image schemata for large-scale space and table-top space are chosen based on a single and restricted type of experience. We investigated both situations methodically and derived the image schemata that are interacting in each type of environment.

The subset of reality considered for large-scale space consists of some geographic-space-objects plus the immediate relations between them. The geographic-space-objects are of the following types:

- LOCATION: This image schema is missing in Johnson's list but seems to be important for geographic space. We use it as a position in space, marked by a named populated place.
- PATH: A PATH connects locations and consists of a starting point, an endpoint and a connection between them, as defined in (Johnson 1987).
- REGION: We use this image schema similar to Johnson's CONTAINER schema. A REGION has an inside, an outside, and a boundary, and represents the idea of containment.
- BOUNDARY: This image schema is similar to Johnson's CENTER-PERIPHERY schema, but BOUNDARY is also part of Johnson's CONTAINER schema description.

The objects in geographic space cannot move and their relations are fixed (but not precisely known). In addition, movable objects such as PERSONs and their location in space are considered. The concrete examples are taken from the Eastern European environment (Figure 1).

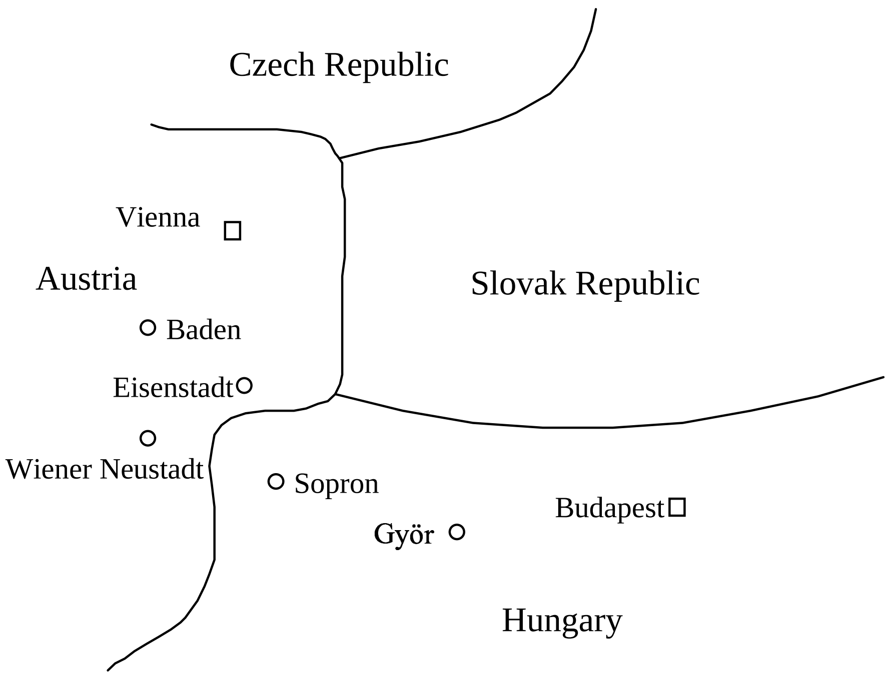

Figure 1: Map of Eastern European Environment.

We first treat the relations between the geographic objects and then the movement of persons between them.

6.1 Base Relations

A scene is represented by a number of facts, which seem to be cognitively salient and basic and not redundant. In particular, we prefer simple relations (i.e., which are partial functions). There is no cognitive justification for these choices of base relations—other relations could be selected. For the scenes considered, we use two simple relations (Bird and Moor 1997), i.e.,

- location in region, and
- region inside region

and two non-simple symmetric relations, i.e.,

- location directly connected by a path to location, and
- region borders region.

For each base relation, a function with two parameters to test the existence of a particular fact and a function with one parameter to return a list of the related values are constructed. Finally, there is a relation indicating where a person is.

6.2 Location and Relations between Places

A path connects places. We differentiate between the simple “direct path” and the “indirect path”, which consists of a sequence of “direct paths.” At this level, different types of paths are not differentiated (i.e., no particulars of railways, highways, etc. are considered).

6.2.1 Direct and Indirect Path

A direct path connects locations directly, without any intervening location (at the level of detail considered). A direct path has a start and an end location (Figure 2a). At this level of detail there is no need to model path as an object, just as a relation between two places (path (a, b)).

Es gibt einen Weg von Wien nach Baden.
There is a path from Vienna to Baden.

For the considered environment (but not for a city with one-way streets) the path relation is symmetric (Figure 2b):

path (a, b) <=> path (b, a)

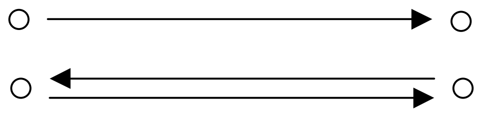

Figure 2a,b: Direct path and symmetry of path relation.

Path is its own converse relation:

Du kannst von Baden nach Wien fahren und am Abend wieder zurück.
You can drive from Baden to Vienna, and back in the evening.

conv (path (a, b)) = path (b, a) = path (a, b)

It is derived from a non-redundant base relation as the symmetric completion.

An indirect (transitive) path (ind-path) connects two locations through a sequence of direct-path-relations, such that the end location of one direct path is the start location of the next path (Figure 3).

ind-path (a,b) = [path (a, a1) & path (a1, a2) & path (a2, ...) & ... & path (..., bn) & path (bn, b)]
conv (ind-path) = ind-path

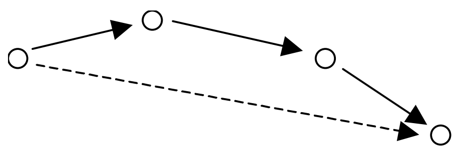

Figure 3: Indirect path.

The indirect path is derived using transitive closure. The details of the algorithm deal with cyclic and bi-directional graphs as formed by path networks and well known as shortest path algorithm (Dijkstra 1959, Sedgewick 1983).

6.2.2 General Connection : "über" or "durch"

Wenn du von Wien nach Budapest fährst, dann fährst du durch Györ. Der Weg von Graz nach Wien führt über Baden und Wiener Neustadt.
If you drive from Vienna to Budapest, you will drive through Györ. The way from Graz to Vienna goes through Baden and Wiener Neustadt.

An indirect path goes "via" its intermediate locations:

ind-path (a to b via c) => path (a, c) & path (c, b)

6.2.3 Detour

A path has a length and generally there are several paths between two locations, some of them shorter than others. The concept of an "Umweg" (detour) is a path that is longer than the shortest path (Figure 4).

Der Weg von Wien über Sopron nach Budapest ist ein Umweg. Der direkte Weg führt über Györ.
The way from Vienna to Budapest through Sopron is a detour. The direct route goes through Györ.

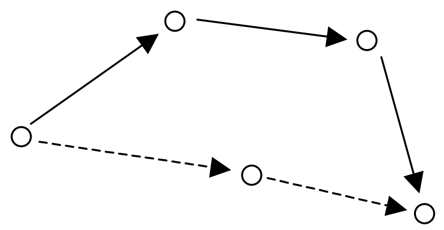

Figure 4: Detour.

A path x from a to b is a detour:

length (ind-path (a to b via d)) < length (ind-path (a to b via c)) => detour (ind-path (a to b via c))

6.3 Relations with Region

Regions are used to represent the idea of containment. They can contain other regions and/or locations.

6.3.1 Region inside Region

A region can be inside another region (asymmetry) (Soja 1971) (Figure 5).

Die Steiermark ist in Österreich.
Styria is in Austria.

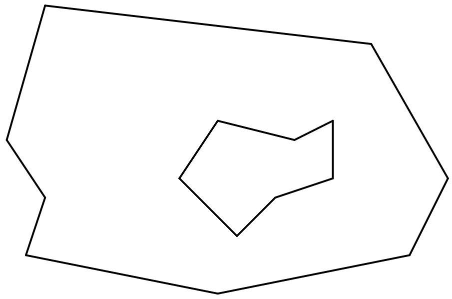

Figure 5: Region inside region.

The converse of inside is contains.

Österreich enthält die Steiermark.
Austria contains Styria.

6.3.2 Indirect inside

Inside for region is transitive: if region1 is in region2 and region2 is in region3, then region1 is indirectly in (*in**) region3 (Figure 6).

in (region1, region3) <=> in (region1, region2) & in (region2, region3)*

Die Steiermark ist in Europa.—(weil Österreich in Europa ist)
Styria is in Europe.—(because Austria is in Europe)

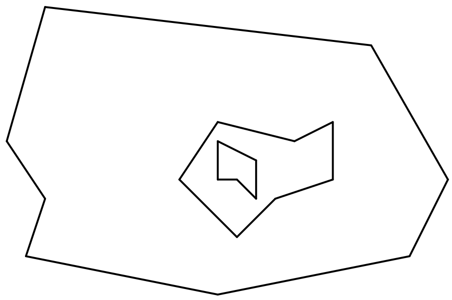

Figure 6: Indirect inside.

One can see that transitivity holds for spatial entities but usually not for legal entities (e.g., Styria is not a member of the EU, although it is a part of Austria and Austria is a member of the EU). Indirect inside is the transitive closure for inside. Indirect contains is the converse.

6.3.3 Location Within Region

Wien ist in Österreich. Graz ist in der Steiermark. Budapest ist in Ungarn.
Vienna is in Austria. Graz is in Styria. Budapest is in Hungary.

If something is within a region and this region is within another region, then the thing is in the enclosing region as well (transitivity of the “in region” relation) (Figure 7).

Graz ist in Österreich.
Graz is in Austria-because Graz is in Styria and Styria is in Austria.

in (loc1, region2) <=> in (loc1, region 1) & in (region1, region 2)*

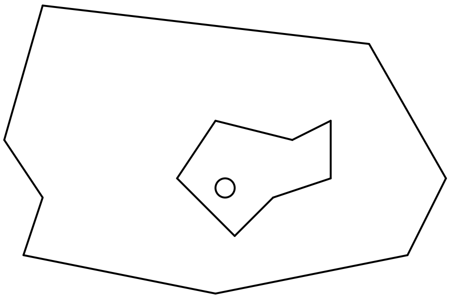

Figure 7: Location indirectly in a region.

The converse is the contains relation for region to all contained locations.

6.4 Relations with Boundaries

Regions have boundaries, which can be conceived as determinate, sharp lines, or one of the different types of indeterminate boundaries (Burrough 1996, Burrough and Frank 1996, Smith 1995).

6.4.1 Neighbor

Ungarn grenzt an Österreich.
Hungary borders upon Austria.

(implies that Austria borders Hungary)
borders (a, b) <=> borders (b, a)

Neighbor is a non-simple (a region can have several neighbors) but symmetric relation (Figure 8). The converse of neighbor is the neighbor relation itself. It is constructed from the non-redundant known relation.

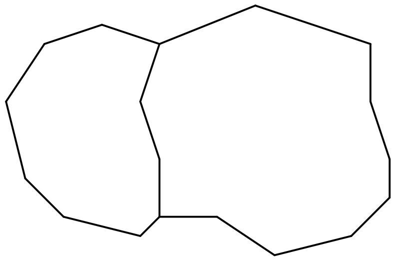

Figure 8: Neighbor.

6.4.2 Island

Das Land Wien ist vollständig von Niederösterreich umgeben. Island ist eine Insel.
The territory of Vienna is completely surrounded by Lower Austria. Iceland is an island.

A region is surrounded by another region (i.e., is an island) if it has only one neighbor (Figure 9).

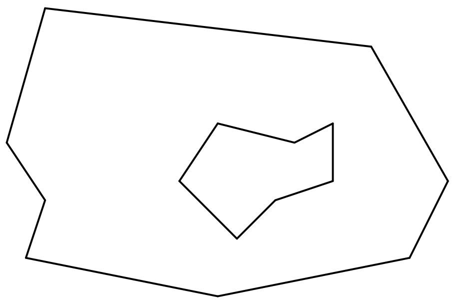

Figure 9: Island.

6.4.3 A Path Crosses a Boundary: Überqueren

If a path leads from a location in one region to a location in another region, it passes a boundary (Figure 10):

Wenn du von Wien nach Budapest fährst, musst du die Grenze in der Nähe von Györ passieren.
If you drive from Vienna to Budapest, you will have to cross the border near Györ.

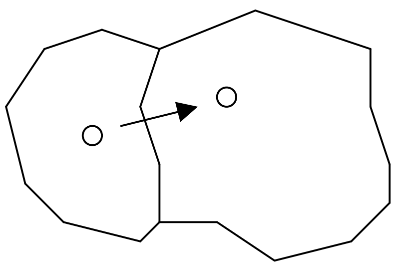

Figure 10: A path crosses a boundary.

The converse of crosses is crossed-by. The relation extends to indirect path.

The issue of the level of boundary—related to the level of the region (county, district, country etc.)—is not considered here.

A path crosses a boundary if its start and end points are not in the same region:

crossesBoundary (a, b) = not (inSameRegion (a, b))

inSameRegion (a, b) = in (a, r) & in (b, r)

This is an application of Jordan's curve theorem: "A simple closed curve (i.e., the topological image of a circle) lying in the plane divides the plane into precisely two regions and forms their common boundary." (Alexandroff 1961 p.2).

6.4.4 Boundary Locations

A location is a boundary location if there is a direct path to a location in another region:

Sopron liegt an der Grenze.
Sopron is at the border.

Exist path (loc a, loc b) & notInSameRegion (loc a, loc b) => OnBoundary (loc a)

A boundary is between two locations if the direct (or indirect) path from one to the other crosses the boundary (Figure 11):

Die Grenze zwischen Ungarn und Österreich liegt zwischen Eisenstadt und Sopron.
The border between Hungary and Austria is between Eisenstadt and Sopron.

Exist path (loc a, loc b) & notInSameRegion (loc a, loc b)=> boundaryBetween (loc a, loc b)
Exist ind-path (loc a, loc b) & notInSameRegion (loc a, loc b)=> boundaryBetween (loc a, loc b)

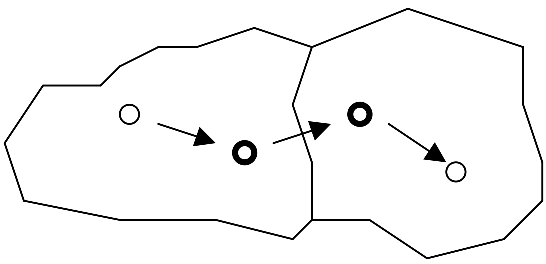

Figure 11: Boundary locations.

6.5 Persons

We now consider relations between persons and geographic objects. In particular, we deal with the movement of persons between geographic objects.

6.5.1 in

Persons are “in” places (Figure 12) and remain there unless they move.

> *Peter ist in Graz. Max ist in der Steiermark, er kann nicht in einem Café in Wien sitzen!*
> *Peter is in Graz. Max is in Styria, he cannot sit in a coffee house in Vienna!*

Figure 12: Person in place.

They can be “in” only one place at a time. The relation is a function from person to location (for each person there is exactly one location); the location may not be known and, therefore, the relation is partial. The converse relation is ”who is ‘in’?”

6.5.2 move

Persons move to places and are then “in” the place, unless they move further:

> *Er ist nach Györ gefahren, jetzt wartet er dort auf dich.*
> *He went to Györ, now he is waiting there for you.*

> *scene2 = move (p, place, scene1) => isIn (p, place, scene2)*

If a person is found “in” place p1 at time t1 and place p2 at time t2 one can deduce a move (Figure 13):

> *Simon war letzte Woche in der Steiermark, jetzt ist er wieder in Wien.—Ist er am Samstag oder am Sonntag nach Hause gefahren?*
> *Last week Simon was in Styria, now he is back in Vienna.—Did he drive home on Saturday or Sunday?—(move inferred in the time in-between)*

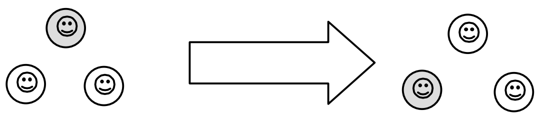

Figure 13: Move.

To move requires for a person some preconditions, unestablishes some facts, and establishes new facts:

> *move (p, a, b): in (p, a) & path (a, b)*
>
> *unestablish (in (p, a)), establish (in (p, b))*

A person cannot move from one place to another unless there is a path:

> *Du kannst von Baden nicht direkt nach Schwechat fahren, du musst über Wien fahren.*
> *You cannot drive directly from Baden to Schwechat, you have to go through Vienna.*

If the person is at an unspecified location within a region, then it is only required that there is a path from every location in this region to the target.

6.5.3 *Persons in Regions*

A person can be at an unspecified location within a region (Figure 14):

> *Er ist in Ungarn auf Urlaub.*
> *He is on vacation in Hungary.*

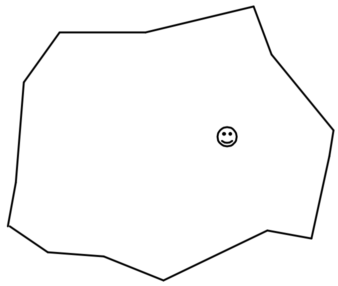

Figure 14: Person in region.

6.5.4 *Deduce "in" Region from "in" Location*

If a person is at a location and the location is inside a region, then the person is in the region (Figure 15):

> *Er ist in Budapest, daher ist er auch in Ungarn.*
> *He is in Budapest, therefore, he's also in Hungary.*
>
> *'in' (X, loc a) & in (loc a, region) => in (X, region)*

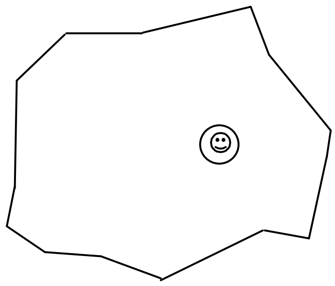

Figure 15: Deduce "in" region from "in" location.

If a person is on a path and the path is in a region, then the person is in the region:

> *Simon ist in Österreich, er ist auf dem Weg von Graz nach Wien.*
> *Simon is in Austria, he is on the way from Graz to Vienna.*

6.5.5 *Position on Path*

> *Er ist auf dem Weg zu dir. Er ist zwischen Wien und Salzburg.*
> *He is on the way to you. He is between Vienna and Salzburg.*

onTheWay (X, a, b) ==> at (X, a, scene 1), path (a, b, scene1), between (X, a, b, scene2), at (X, b, scene3),

Exist a: arrives (X, b) ==> previousOnTheWay (X, a, b)

This is a hierarchical decomposition of a single move in two steps, to leave and to arrive—it is not further considered here.

6.6 Checks for Inconsistencies

The set of base relations contains simple relations induced through the rules of the image schemata. Nevertheless, inconsistencies can be introduced. A person cannot be

- at a location in region a and in region b at the same time;
- in a region a and on a path that is not (at least partially) inside a at the same time.

In a formal model, guards against the introduction of such inconsistencies can be built in; in the database literature these are known as consistency constraints (Date 1986).

6.7 Formal Executable Model

A formal, executable model for the relations presented here has been written in a functional programming language. If a suitable set of support operations to deal with relations in an environment is available, the content of the image schemata is expressed in about 60 lines of code.

The difficulties of coding have mostly to do with finding consistent conventions to name all the relations. If the operations are written “curried”, then most rules can be written as equations between relations and relation transforming functions (i.e., point-free in the categorical sense (Bird and Moor 1997)), and in nearly all the scenes the last argument can be dropped, indicating the formulae are valid for any scene.

The use of a typed relation calculus with polymorphism allows to overload relation names; for example “a location in a region” and “a region in a region” can be reduced to a polymorphic in:: a -> b -> Bool (with two type variables a and b) and instantiations in: Location -> Region -> Bool and in: Region -> Region -> Bool. This is not only “syntactic sugar”, but forces a restructuring of code following the types of the objects related and leads to the identification of commonality. If this code is integrated with the code for relations in table-top (small-scale) space, then the assumption of polysemy can be given up if it is not justified.

7 Formal Specifications of Image Schemata for Small-Scale Space

Based on a single type of experience we consider the following image schemata for small-scale (table-top) space:

- CONTAINER: The CONTAINER schema is similar to the LOCATION or REGION schemata considered for geographic space. A CONTAINER has an inside, an outside, and a boundary.
- SURFACE: The SURFACE schema is used to describe the support of objects.
- LINK: People relate connected objects through the LINK schema.

We focus on the common-sense spatial reasoning conclusions from the relations "in" (CONTAINER), "auf" (SURFACE), and "an" (LINK) between an object and a relatum, and the operations to establish such relations (*moveIn, moveAuf, moveAn*).

Table-top space, as restricted here, is simpler (see assumptions in Section 7.1) than the geographic space treated in section 6. It is, therefore, possible to structure the formalization more and to arrive at a list of uniform conditions (rules 7.2 to 7.5) for the semantics of the prepositions considered.

7.1 Closed World Assumptions

The scene is a tabletop of unspecified objects, which are moved from the outside. The following complete list of assumptions holds:

1. In this world objects can be moved, unless the move is blocked by the relation in which the object participates in.
2. An object can be moved to (in, auf, an) a target, unless access to this target is blocked by a relation in which this target participates in.
3. Every object can enter in any relation with any other object, i.e., all objects can serve as containers or surfaces; objects are not differentiated.

For our restricted type of experience we assume a closed world: Moves are not blocked by other considerations. This implies the following "closure" axioms:

4. No other objects exist.
5. The number or size of other objects which can be related to an object is not limited. Moves are not blocked by size considerations. For example, objects too large for container, surface completely covered, etc. (The ontological study of a room space done by Egenhofer and Rodríguez (forthcoming) considers the relative size of objects in small-scale space.)

7.2 “In” Blocks Target of Movement

An object cannot be moved to a target if this is already in another object (Figure 16). This is justified by situations as:

x 'in' y (in scene) => blocked (move z into x (in scene))

Du musst den Beutel zuerst aus der Tasche nehmen, bevor du die Münze hineingeben kannst.
You must take the purse out of the pocket to put the coin in.

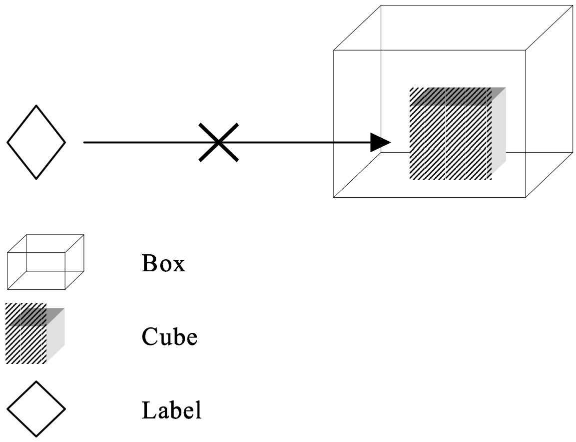

Figure 16: Cube is in box. Not permitted: paste label on cube.

7.3 Converse of “auf” Blocks Object of Movement

“Auf” blocks the movement of the supporting object (Figure 17). It cannot be moved unless the object on (“auf”) it is removed.

x 'auf' y (in scene) => blocked (move y in scene)

Teller und Gläser sind auf dem Tisch. Wir müssen den Tisch zuerst abräumen,
bevor wir ihn auf die andere Seite des Zimmers bringen können.
Plates and glasses are on the table. We have to remove all objects from the table,
before we can move it to the other side of the room.

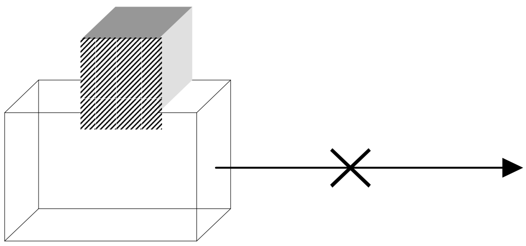

Figure 17: Cube is on (auf) box. Not possible: move box.

7.4 “In” and “an” Block Movement of Object

“In” and “an” create a link between the object and the relatum which resists movement (a particular “break link” operation would be required to break it: unglue, takeOut etc.) (Figure 18).

x 'in' y (in scene) => blocked (horizontal move x in scene)

Der Apfel kann nicht aus der Schale rollen, aber du kannst ihn herausheben.
The apple cannot roll out of the bowl, but you can take it out.

x 'in' y and 'closed' y (in scene) => blocked (move x in scene)

Du musst die Büchse öffnen, dann kannst du die Würfel herausnehmen.
You must open the box. Then you can take out the dice.

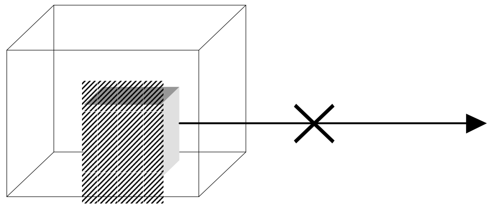

Figure 18: Cube is in box. Not possible: move cube.

“An” presupposes a physical connection between the object and the relatum (stronger and more permanent than gravity support) which is typically established intentionally (verbs like to nail, to glue, to stick, etc. and not just plain “to put”). “An” with this definition is seen as a LINK image schema. Movement is restricted unless the link is broken (Figure 19).

x 'an' y (in scene) => blocked (move x in scene)

Ich habe das Papier auf das Buch gelegt, jetzt klebt es daran.
Wenn du das Papier mitnehmen willst, musst du es sorgfältig lösen.
I put the paper on (auf) the book, now it is glued on (an).
If you want to take it with you, then you have to carefully remove it.

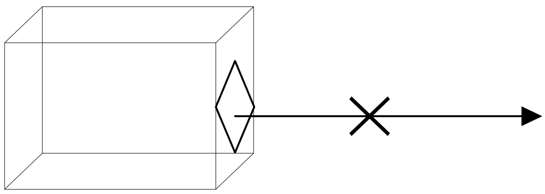

Figure 19: Label is glued to box. Not possible: move label.

7.5 “In” and “an”: Invariance Under Movement of Relatum

Corresponding to the blocked access to the object for “in” and “an” relations (rule 7.4), these relations are invariant under movement (Figures 20, 21). If x is “in” y and y is moved, then x is still “in” y (and the same for “an”).

Du hast das Buch im Büro in die Tasche gegeben. Jetzt bist du in meiner Wohnung und kannst es herausnehmen.
You put the book into the bag at the office. Now you are at my apartment and you can take it out.

x 'in' y (in scene) => x 'in' y (in move y in scene) = True

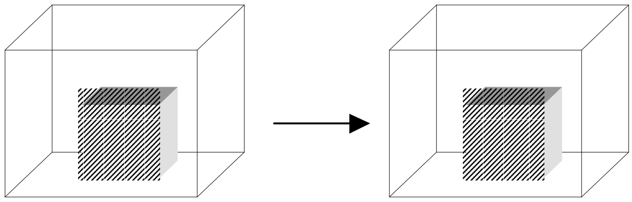

Figure 20: Cube is in box. Invariant under: move box.

x 'an' y (in scene) => x 'an' y (in move y in scene) = True

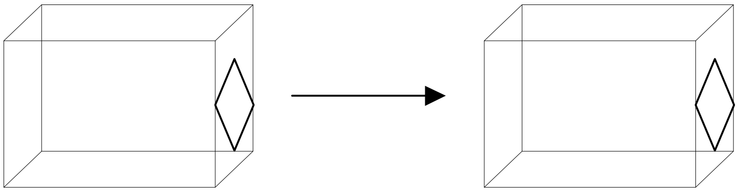

Figure 21: Label is glued to box. Invariant under: move box.

These rules will not be expressed explicitly, as they are subsumed by the “stable world property” (nothing changes unless specifically indicated). Without such restriction one would have to consider the frame problem (Reiter 1984).

7.6 Undoes a Previous Relation of Object: “Auf”

“Auf” does not restrict the movement of the object (Figure 22):

x 'auf' y (in scene) => move x in scene

Du kannst das gelbe Buch nehmen, es liegt auf dem Tisch.
You can take the yellow book, it is on top of the table.

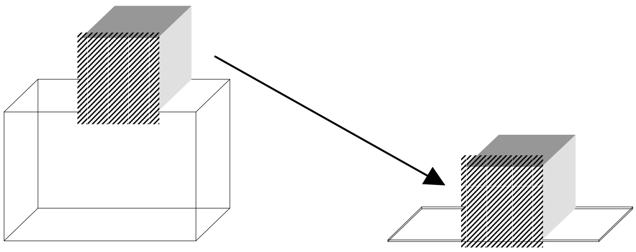

Figure 22: Cube is on box. => Cube is on table. Cube is not on box.

The effect is, however, that the previously established relation is false and a new relation is established:

move x ‘auf’ y (scene1, t1) => scene2, t2

x ‘auf’ z (in scene1) = True

x ‘auf’ y (in scene1) = False

x ‘auf’ z (in scene2) = False

x ‘auf’ y (in scene2) = True

7.7 Formal Model

A function composition model can be constructed and the rules listed are directly coded. The central operations "move" with the arguments *relation type*, *object*, *target*, and *scene* are shown below; the complete program is only 31 lines of code! This formal model represents the axioms from Section 7 as a sequence of conditions.

```
move i a b s =
        if fRel' In b s          -- rule 7.2 : in blocks target of movement
                then error ("in blocked: already in")
        else
        if fRelConv Auf a s      -- rule 7.3: (conv auf) blocks movement
                then error ("auf move blocked: already covered")
        else
        if fRel In a s           -- rule 7.4 (1): in blocks movement of object
                then error ("in move blocked: already in")
        else
        if fRel An a s           -- rule 7.4 (2): an blocks movement of object
                then error ("an move blocked: obj already an")
        else
        if fRel Auf a s          -- rule 7.6: undoes previous 'auf' of object
                then move i a b (takeOff Auf a s)
        else
        establish i a b s
```

Compared to Section 6.5.2 (*move* in large-scale space) where we only had to check the existence of a path, *move* in small-scale space can be restricted for a number of reasons (e.g., blockage), therefore the formalism contains more conditions.

8 Conclusions, Open Questions, and Future Work

Formal descriptions of spatial relations as encountered in everyday life are very important for GIS. They can be used to formally define query language predicates and to optimize the execution of spatial queries. They are crucial for the specification of spatial data exchange formats and GIS interoperability standards.

Most previous efforts to analyze spatial relations have used relation calculus and have concentrated on spatial relations which are amenable to this treatment. The extension of relation calculus to a function calculus is discussed here, linking two previously unconnected tools. The two tools are not as different and their conceptual merging is in category theory (Barr and Wells 1990, Herring *et al.* 1990, Asperti and Longo 1991, Walters 1991). Function composition tables can be used similarly to relation composition tables; they show patterns which can then be succinctly formulated as rules.

In this paper we applied a linguistic method based on prepositions to describe image schemata. A rich set of relations for 4 geographic-space-image-schemata (i.e., LOCATION, PATH, REGION, and BOUNDARY) was presented in a formal way. From 5 base relations we deduced around 15 meaningful relations (not counting the corresponding converse relations). With this approach the common-sense knowledge about the environment considered is captured in a strong set of implications following from individual relations. In this domain most of the relations are static and geographic objects do not move, only people move among them (a key concept in the definition of geographic space). It may be surprising how much deduction is actually possible at this high level of abstraction, where neither form nor location of individual objects are considered. Furthermore, the method was applied to 3 table-top-image-schemata (i.e., CONTAINER, SURFACE, and LINK). Formal definitions were given based on 3 types of rules. The differentiation between “an” and “auf” in German seems not to depend on vertical vs. horizontal surface, but of linkage between the object and the relatum (i.e., gravity vs. physical attachment).

Both domains are very powerful as a source for metaphors. For each of the concrete usages given here a corresponding metaphorical usage can be suggested (Lakoff and Johnson 1980, Lakoff 1987, Johnson 1987). Geographic space is typically used to structure the space of ideas—one could posit an overarching metaphor “the world of ideas is like geographic space”: ideas are connected (by logical paths), people have arrived at some position, but not yet moved on to a new understanding, in order to move from one camp (political party) to another, one has to cross a boundary… This corresponds well to the “life is a journey” metaphor (Lakoff 1987, Johnson 1987) where the journey is used to structure a large number of aspects of our understanding of our lives.

Many open questions still remain and should be considered for further research:

8.1 Methodological

The method used here is borrowed from linguistics. Therefore, it has to be seen if such an approach may be limited because of its dependence on language. More research will be necessary to find out about the limitations of linguistic methods for studying human cognition. For linguistic demonstrations, a single utterance which is acceptable by a native speaker is sufficient to demonstrate the existence of a construct. Is a single commonsense reasoning chain as given here sufficient? It documents that at least a situation exists where the suggested spatial inference is made—thus it demonstrates at least one aspect of a spatial relation in (one human’s) cognition.

In order to verify the universality of such spatial inference mechanisms, extended human subjects testing among people with different native languages is needed.

8.2 Language-Independent Primitives

Can language-independent primitives be identified (in the sense of Wierzbicka (1996))? Specifically for English and the table-top examples discussed: is "on" a single word in English or is a polysemous definition better (with the two meanings of German "an" and "auf")? Are the indirect forms ("indirekt in" and "indirekt auf") different from the direct forms (polysemy)? There are cases when the two are differentiated:

> *Ich sitze nicht auf dem Boden, ich habe mir eine Zeitung untergelegt.*
> *I do not sit on the ground, I have put a newspaper under me.*

Investigation of the same domain by researchers with different mother tongues would be necessary (or at least a collection of the related natural language descriptions). For the domains and examples here, the spatial inferences are also correct in the translations, but the use of spatial prepositions differ between German and English.

8.3 Connection Between Relations and Functions

The use of category theory to establish a common theoretical ground for a relation (static) view and a function (dynamic) view is new and must be further explored. A category can be constructed over both functions and relations (Bird and Moor 1997). It is also possible to map relations into simple functions (aRb --> f (a,b) : Bool) and functions into a relation (f (a):: b --> aRb) as was used here. Certain formalizations seem to be easier using relations, others using functions.

In any case, the formulae must be interpreted with respect to an "environment" of the facts (we used the term "scene"). Functions like "move" change the scene. We currently experiment with monads—a device from category theory—to have the environment implicit in the formulae and, therefore, reduce the complexity of formalization (Wadler 1997, Liang *et al.* 1995).

8.4 Composition and Interaction of Image Schemata

The combination of multiple image schemata and the interaction of image schemata with object properties must be further explored. For an object to move along a path, it must be of the appropriate kind (only trains run along railway lines, cars cannot follow a foot path, etc., and similar restrictions apply in other cases). Possibly, the current approach trying to capture image schemata with the definition of spatial prepositions is too limited. Raubal *et al.* (1997) used prepositions and semantic connotation to investigate superimpositions of image schemata. Another interesting approach is to look at affordances. Affordances seem to be closely related to image schemata because both of these concepts help people understand a spatial situation in order to know what to do (Gibson 1979). Affordances might be operational building blocks of image schemata but further research in this area is needed (Jordan *et al.* 1998).

Type theory as used in today's advanced programming languages (Jones 1994) provides a flexible framework that could capture the category structure of subcategories and their interaction with image schemata, but further work is necessary.

8.5 Comparison with the Modeling of Other Domains and Integration of Image Schemata Across Domains

Our model of geographic-space-image-schemata must be extended with other models, e.g., the environment of a journey (path, roads, junctions, etc.) or a city scape (Lynch 1960). If these image schemata are formally described and the interaction between image schemata and the category structure is clear, integrated models can be achieved, parallels identified, and duplication removed.

8.6 Are Image Schemata the Smallest Constituent Parts of Spatial Cognition?

Are image schemata the atoms of spatial cognition or are there smaller semantic units from which image schemata can be composed? It appears as if there were smaller pieces from which the more complex image schemata could be built (especially along the lines of the rules 7.2 to 7.6), but one could also argue that these are the image schemata proper.

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